Conceptions of freedom and ranking opportunity sets. A typology

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Abstract

A wide diversity of rankings of opportunity sets are characterized through what is now commonly called the freedom of choice literature. We claim the normative content of each of these propositions can be analyzed and clarified through a typology. We distinguish two kinds of rankings: rankings according to one prudential value and rankings according to several prudential values. In the first case, we present rankings according to diverse prudential values such as freedom of choice, freedom as autonomy, freedom as exercice of significant choices, negative freedom, positive freedom and utility. In the second case, the rankings may capture some specific notions of overall well-being. We organize the presentation around different forms of commensurability between prudential values: weighting, trumping, equal consideration and discontinuity.

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1 Introduction

The literature of ranking opportunity sets (see Barberà, Bossert and Pattanaik (2001)), tackles a wide range of problems such as choosing in an uncertain environment or valuing freedom of choice *per se*. This article focuses on the normative side of this literature, now standardly called the freedom of choice literature and in which the range and/or the content of opportunity sets is at stake.

To define what an opportunity set is, one can represent individual choice as a two stage problem. At early stage, individual makes decisions which will constraint the scope of later feasible choices over options. This amounts to pick one set of options among many, or, at least, rank sets of options. A later stages, individual will choose one option out of the previous set. Let's say an opportunity set is a restaurant menu: you first choose the restaurant among all available restaurant in the city, then you pick a meal within the chosen restaurant's menu. The aim of the freedom of choice literature is to rank opportunity sets according to some normative criterium such as utility, freedom of choice, any notions of freedom, individual overall well-being.

Let X be the universal set of alternatives. Let $x, y, ... \in X$ be the alternatives (commodities, options, actions...) the agent may be faced at second stage. Let $\sharp X$ be the cardinal of the set X. As in the traditional microeconomic framework, individuals may have preferences over the alternatives. Let $\mathbf{R}$ a reflexive and transitive binary relation over X. We may call it a preorder. $x\mathbf{R}y$ means that the option x is at least considered as good as y according to preferences $\mathbf{R}$. The symmetric and asymmetric part of $\mathbf{R}$ are respectively denoted $\mathbf{I}$ and $\mathbf{P}$. Let $\Pi(X)$ the power set of X, i.e. the set of all non-empty subsets of X. The elements of $\Pi(X)$, denoted as $A, B, ...$ are the opportunity sets the agent may be faced in the first stage. Let $\sharp A$ be the cardinal of the set A. Let $\succeq$ be a reflexive and transitive binary relation defined over $\Pi(X)$. We may call it a preorder. For all $A, B \in \Pi(X)$, $(A \succeq B)$ is to be interpreted as "the set A is considered at least equally good than B according to the specific value at stake or the retained conception of well-being". It could mean, for instance: 'A provides at least as much freedom as B'. The symmetric part and assymetric part of $\succeq$ are respectively $\sim$ and $\succ$. In the freedom of choice literature, the preorder $\succeq$ is to be characterized axiomatically, i.e. the set of basic conditions under which a specific ranking of opportunity sets hold is specified.

The presentation of the freedom of choice literature in this paper is not intended to be an exhaustive survey[1]. I aim at proposing a typology of the formal rankings displayed in this literature, according to some specified prudential value or combination of values. The typology relies on the study of the different kinds of conditions characterizing the results: normative criteria, state description. Each ranking is eventually defined by a combination of what I will call here 'conditions', that are often called 'axioms' or 'definitions' in the literature. Each of these conditions and their combination contribute to capture some specific prudential value, such as freedom or utility or specific notions of overall well-being[2]. In particular, the richness of this literature yields to distinctions between different notions of freedom, nay between different notions of freedom of choice. Besides, some rankings do not concern just one specific prudential value but combine several values, inducing individual overall well-being rankings. Though this work should not be taken as a typology of all different conceptions of freedom. Indeed, the chosen framework restricts the scope of possible meanings. On the one side, as we focus on individual rankings only, we cannot discuss the social aspects of freedom (See Bavetta (2004), Carter (2004), Oppenheim (1995), (2004)). On the other side, as we focus on opportunity sets, we cannot discuss political liberties or freedom of the will (See Hayek (1960: 11–21)).

In this paper, we will firstly present results concerning rankings of opportunity sets according to one prudential value (section 2). We will distinguish between freedom of choice as opportunity, freedom of choice as autonomy, the valuation of exercice of choice, negative freedom, positive freedom, and utility. We will secondly display rankings of opportunity sets according to conceptions of overall well-being (section 3). We will distinguish between different forms of commensurability: weighting values, trumping, equal consideration of different values, and discontinuity.

[1] To our knowledge, there does not exist any specific survey on the freedom of choice literature. For a general survey on the rankings of opportunity sets, see Barberá, Bossert and Pattanaik (2001). For a critical presentation of the freedom of choice literature, see Bavetta (1995), (2004). For a survey of a close literature, see Peragine (1999) .

[2] Other interpretations than those we will display in the article for each contribution are certainly possible. We do not pretend to close any discussions about the correct interpretations. See, for instance, the discussion of the concept of freedom under the cardinal ranking by I. Carter (2004).

2 Rankings of opportunity sets according to some specific value

Most rankings of opportunity sets pretend – and actually correspond – to some judgements according to freedom of choice. But we here claim this could actually be more specified: freedom of choice is not a narrow concept, it does allow many different meanings such as an opportunity conception of freedom – freedom as scope of choice or autonomy – or an exercice conception – the valuation of exercice of choice *per se*. Besides, we will notice that freedom of choice is not the only prudential value likely to be captured through rankings of opportunity sets. Distinct noticeable concepts of freedom or other values are also represented in this literature, such as negative freedom, positive freedom or utility. I do not present each prudential value in details, I rather show how the formal framework and the specific characterization shape the concept and define a particular meaning of freedom of choice or other values.

2.1 The scope of choice and freedom of choice

The 'freedom of choice literature' was basically initiated by the seminal article of P. Pattanaik and of Y. Xu (1990), published in *Recherches Économiques de Louvain*: 'On ranking opportunity sets in terms of freedom of choice'. The idea of freedom of choice is captured by the combination of three conditions: indifference to no-choice situations, independence and simple monotonicity.

A way to express the value of freedom of choice is to value the fact that there is choice (See Carter (2004: 72)). Presenting no-choice situations is likely to capture the idea of no-choice.There is indeed no reason why one singleton would provide more choice than another singleton as far as there is no choice in each. Here the difference between ranking according to choice or freedom of choice on the one side and utility on the other side is made very clear. If utility was at stake, the two singletons would not be indifferent but their ranking would be based on the preference of the options.

Condition 1 (Indifference between no-choice situations) $\forall x, y \in X$, $\{x\} \sim \{y\}$.

Besides, the intrinsic importance of freedom of choice, rather than its instrumental value, is defined by Ian Carter (1999: 41) in following terms: 'One phenomenon X has an intrinsic value if and only if X is one end in itself, i.e., if X has a positive total value which is not not reducible with the value of any other phenomenon'. He proposes a test to determine whether or not an intrinsic importance to freedom of choice is expressed in some judgment. Let two situations S_1 and S_2. S_2 is composed by the options contained in S_1 and eight others. The preferred options are already included in S_1. One eventually must choose only one option. Which of the two sets seem to be more valuable? If I find S_2 more valuable, freedom has an intrinsic importance to me. In indeed find valuable options that I will choose but also the others: I therefore find valuable to be able to push back the nine other options. These nine 'not' gives a direction to my life, they reveal my identity through the actual choices in my life. More choice is thus always considered better than less choice, whatever the options. The valuation of 'more choice' or 'more freedom of choice' *per se* then requires a strict monotonicity condition.

Condition 2 (Strict monotonicity) $\forall x, y \in X, x \neq y, \{x, y\} \succ \{y\}$.

A third condition is necessary to induce a ranking. Pattanaik and Xu propose an independence condition, requiring that it A and B are two possible available sets and if x does not belong either to A or to B, then the ranking of A and B in terms of freedom[3] corresponds to the ranking of $A \cup \{x\}$ and $B \cup \{x\}$.

Condition 3 (Independence) $\forall A, B \in \Pi(X), \forall x \in X \setminus (A \cup B)$,
$[A \succeq B$ *if and only if* $A \cup \{x\} \succeq B \cup \{x\}]$.

According to the cardinal ranking[4], the more options in a set, the more choice is provided by the set. The scope of choice is simply assessed by counting the number of options contained in the set.

Rule 1 (Cardinal ranking) *For all* $A, B \in \Pi(X)$*,* $A \succeq B$ *if and only if* $\#A \geqslant \#B$.

Proposition 1 (Pattanaik et Xu (1990)) *Preorder* $\succeq$ *satisfies simple monotonicity (condition 2), indifference between no choice situations (condition 1) and independence (condition 3) if and only if* $\succeq$ *is the cardinal ranking.*

The cardinal ranking is characterized by conditions capturing the ideas of freedom, choice and freedom of choice. Nevertheless, the simpleness of this results gave rise to many constructive critics and, eventually, to many sophisticated formulations of the expression of freedom.

[3]The fact that this version of the independence condition captures the idea of freedom of choice is controversial. See Carter (2004).

[4]For a presentation and justification of such a rule, see Jones et Sugden (1982), Sen (1985), and van Hees (1998).

2.2 Reasonable preferences and autonomy

An individual is autonomous if she makes choices entirely according to her will, meaning independently of her conditionings or other people's will.

P. Jones and R. Sugden (1982) raise a tension between the economic theory of choice according to which the preferences are given, such as the standard framework of microeconomic and the valuation of autonomy. Let's take the standard representation of choice in economics, relying on the strong model of preferences (see below). Whenever Mister A chooses x rather than y among the opportunity set $\{x, y\}$, he reveals he does prefers x rather than y. For mister A's given preferences, x is a better option. Let's now imagine a second case : mister A is given the option x from the opportunity set $\{x, y\}$ by force, coercion, pity, in each case, without even asking him what he desired. Mister A derives the exact same satisfaction in both cases since he will anyways get his preferred option. Jones and Sugden (1982: 59) therefore propose to challenge the assumption of given preferences to avoid this annoying consequence.

> 'To suppose that the act of choice requires the exercise of mental powers is to suppose that the chooser is in some considerable measure an autonomous agent; whatever he chooses, he might have chosen something else. There is a tension between those and the idea implied in the economic theory of choice, that preferences are given. What makes significant choice possible is that preferences are not just part of a person's physiology or psychology like the color of his eyes or a tendency to depression. [...] The concept of significant choice can best be understood by considering the various preferences that a person might have, rather than merely the preferences that he actually reveals when he finally makes a choice.'

Preferences do only catch the idea of the satisfaction he could get from different options, whatever chosen or imposed. Their account is then not likely to capture the idea of autonomy. In the example, Mister A is definitely more autonomous in the first case than in the second one. His choice is indeed autonomous if he chooses something while he could have chosen something else. He is autonomous if his preference relation could actually have been different. The account for other possible preferences excludes the cases where he is obliged to get his actual preferred option. Therefore, the introduction of a wide scope of possible preferences in the assessment of opportunity sets, which we will now call 'reasonable preferences', allows to capture the idea of autonomy: the wider the range of reasonably chosen options from an opportunity set is, the more autonomous the person who faces it is.

Following Jones and Sugden (1982) and Puppe (1998), the options that are likely to be chosen, according to reasonable preferences[5], will be called essential options. Denote $E(A)$ the subset of essential alternatives from the set A. This also means that an option is essential if extracting it from the set strictly decrease the freedom derived from a set: $E(A) := \{x \in A : A \succ A \backslash \{x\}\}$.

On the contrary, an option is ineligible if it could never be chosen, e.g. by any reasonable preference. The fact that certain option may never be chosen have an important consequence on the formulation of monotonicity condition. If you add another option to a set, you can no longer consider that autonomy is raised no matter what option it is. Another option will never decrease the autonomy provided by the set, but it is likely not to raise it if it is ineligible. Therefore, while we had justified the appeal to a strict monotonicity condition for the choice ranking, the autonomy ranking should respect a weak monotonicity condition.

Condition 4 (Weak monotonicity) *For all* $x, y \in X$*,* $\{x, y\} \succeq \{x\}$.

The kind of further alternatives likely to raise the autonomy derived from a set has to be specified. Another monotonicity condition, called I-monotonicity and proposed by Pattanaik and Xu (1998), captures the idea according to which only essential alternatives may increase the autonomy provided by an opportunity set. Let's first introduce some notations. Considering all reasonable preferences to identify the best elements, for all $x \in X$ and all $A \in Z$, $x[\mathbf{I}]A$ if and only if $max(A \cup \{x\}) = A \cup \{x\}$; $x[\mathbf{P}]A$ if an only if $\{x\} = \max(A \cup \{x\})$; $A[\mathbf{P}]x$ if and only if $x \notin \max(A \cup \{x\})$.

Condition 5 ((I)-monotonicity) *For all* $A, B \in \Pi(X)$*, and for all* $x \in X \setminus A$*,* $(x[\mathbf{I}]A$ *and* $A \succeq B)$ *implies* $[A \cup \{x\} \succ B]$.

The assessment of autonomy will be different according to whether a broad range of possible preferences is considered as reasonable or very few of them. There are two kinds of differences : on the nature of restrictions of the set of

[5]Let's remark it is equivalent, in the opportunity set framework, to think in terms of possibly chosen option in a set or in term of possible preference. As a matter of fact, all options are considered at first to pick one single option subsequently, option that is the best element according to a preference relation. If there are several possible preference relations, there might be several possible chosen options. Henceforth, we will considered that preference relations differ at least on their best elements to make the reasoning perfectly equivalent.

possible preferences to a set of reasonable preferences on the one hand, on the design of such restrictions on the other hand.

2.2.1 All rational preferences are reasonable: Valuing the scope of choice

To one extreme, all possible rational preference may be considered as reasonable. Even very odd and eccentric preferences should be accepted as reasonable. J. S. Mill (1859: 83) for instance would defend this idea according to which there is no justification to restrict the scope of reasonable preferences. Therefore, each option from a set is essential, no matter which it is. The required monotonicity condition here is therefore exactly equivalent to the strict monotonicity condition 2. We are then brought back to the cardinal ranking of the opportunity sets, e.i. to Pattanaik and Xu (1990)'s result.

2.2.2 Just one preference ordering is reasonable: Valuing indirect utility

To an other extreme, if just one preference is considered as reasonable, the actual individual preference, then we are back to the traditional rule based on the comparison of indirect utilities.

Rule 2 *For all* $A, B \in \Pi(X)$*,* $A \succeq B$ *if an only if* $[max(A) R \max(B)]$.

2.2.3 Some preferences are reasonable, some are not: Valuing autonomy

Restrictions on the family of the acceptable preferences may be justified. For instance, we could claim that no reasonable person may ever consider that being beheaded at dawn is an essential option. Though to our knowledge not much in the freedom of choice literature describes formally these different kinds of preferences. Rather, we focus on the design of the restriction, specifying whether or not the fact that an option is essential is given, thus capturing different conceptions of autonomy. We display the rule based on the cardinality of essential options, on the cardinality of essential options weighted by ineligible options, then the lexicographic version of these rules.

Simple cardinality of essential options If, for all reasonable preferences, x is strictly worse than at least one option from A, e.g if x is ineligible, then adding x to A does not raise autonomy.

Condition 6 (Type 1 irrelevance of dominated alternatives) *For all* $A, B \in \Pi(X)$ *and for all* $x \in X$*, if* $A[\mathbf{P}]x$*, then* $[A \succeq B$ *if and only if* $A \cup \{x\} \succeq B]$ *and* $[B \succeq A$ *if and only if* $B \succeq A \cup \{x\}]$*.*

The basic composition condition, corresponding to an extension of the independence condition to inclusions of any sets rather than a singleton, is the following: for all $A, B, C \in \Pi(X)$, we have : $A \sim A \cup B$ implies that, for all $C, A \cup C \sim A \cup B \cup C$. If $A \succeq B$ and $C \succeq D$, we could expect that $A \cup C \succeq B \cup D$. This formulation is not used in this setting for it is likely to induce counter-examples. If A and C share too numerous common elements, it is possible than $B \cup D$ provides less freedom nevertheless. The composition condition proposed by Sen (1991) is designed to avoid this annoying consequence : we further assume that the sets do not have common elements, e.g. $A \cap C = B \cap D = \varnothing$. But even this new formulation is likely to induce counter-examples in this settings for some alternatives from the sets could be ineligible. The specific composition condition proposed by Pattanaik and Xu (1998) requires then that each distinct alternative from a set is an essential alternative.

Condition 7 (Composition) *For all* $A, B, C, D \in \Pi(X)$*, such that* $(A \cap C = B \cap D = \emptyset$*, and* $\max(A \cup C) = A \cup C$*, and* $\max(B \cup D) = B \cup D)[A \succeq B$ *and* $C \succeq D]$ *implies* $[A \cup C \succeq B \cup D]$*, and* $[A \succeq B$ *and* $C \succ D]$ *implies* $[A \cup C \succ B \cup D]$*.*

Cardinal ranking of essential alternatives compares opportunity sets according to the number of essential alternatives in the sets.

Rule 3 (Cardinal ranking of essential alternatives) *For all* $A, B \in \Pi(X), [A \succeq B$ *if and only if* $\# \max(A) \geqslant \# \max(B)]$ *, where* $\# \max(A)$ *stands for the number of time a reasonable preference considers an element from* A *is optimal.*

Proposition 2 (Pattanaik et Xu (1998)) *The relation* $\succeq$ *satisfies indifference between no choice situations (condition 1), I-monotonicity (condition 5), type 1 irrelevance of dominated alternatives (condition 6), and composition (condition 7) if and only if* $\succeq$ *is the cardinal ranking of essential alternatives.*

Let's take the following example. We consider two sets: $A = \{x, y\}$ and $B = \{z, w\}$ and two possible reasonable orderings of preferences: $x\mathbf{P}y\mathbf{P}z\mathbf{P}w$ and $y\mathbf{P}'x\mathbf{P}'w\mathbf{P}'z$. Therefore, the set of essential elements from A is $\{x, y\}$

and from B is $\{z, w\}$. In both cases, the cardinal of essential elements is the same, which induces that they provide exactly the same autonomy according to the rule 3. It is quite difficult to swallow when considering than a reasonable person would never choose an option from B if he was given the choice to pick one from A as well. Intuitively, the fact that some elements are more likely to be chosen is important, not just the fact that they are likely to be chosen in some set.

Weighted cardinality rule of essential options Another form of restriction consists in taking into account the role of ineligible actions in the set.

We first formulate dominance and non-dominance conditions in a similar framework.

Condition 8 (Simple non-dominance) *For all $x, y \in X$, if $x[I]y$, then $\{x\} \sim \{y\}$.*

Condition 9 (Simple dominance) *For all $x, y \in X$, if $x[P]y$, then $\{x\} \succ \{y\}$.*

If, for all reasonable preferences, x is strictly worse than at least one option from A, i.e. if x is ineligible, then the status of the set $B \cap \{x\}$ vis-à-vis A should not be better nor worse that the status of B vis-à-vis A.

Condition 10 (Type 2 irrelevance of dominated alternatives) *For all $A, B \in \Pi(X)$ and for all $x \in X$, if $A[\mathbf{P}]x$, then $[A \succeq B$ if and only if $A \succeq B \cup \{x\}]$ and $[B \succeq A$ if and only if $B \cup \{x\} \succeq A]$.*

A weaker composition condition adds some further conditions to the composition condition 7: every alternative from $A \cap D$ can be considered by a reasonable person to be at least as good as all alternatives in $A \cap D$, and every alternative in $B \cap C$ can be considered by a reasonable person to be at least as good as all alternatives in $B \cap C$.

Condition 11 (Weak composition) *For all $A, B, C, D \in \Pi(X)$, such that $(A \cap C = B \cap D = \emptyset$, and $\max(A \cup C) = A \cup C$, and $\max(A \cup D) = A \cup D$, and $\max(B \cup C) = B \cup C$, and $\max(B \cup D) = B \cup D)$, $[A \succeq B$ and $C \succeq D]$ implies $[A \cup C \succeq A \cup D]$, and $[A \succeq B$ and $C \succ D]$ implies $[A \cup C \succ B \cup D]$.*

Let A^B refers for all elements $a \in A$ such that reasonable person may never consider a to be at least as good as all the elements of B. Of course, these elements are not necessarily identical with B^A. P. Pattanaik and Y. Xu (1998) characterize a weighted cardinality rule of essential options. In this ranking of different sets, what is at stake is not just the number of essential alternatives in each sets but also the fact that these sets lead to valuable choices.

Rule 4 (Weighted cardinal ranking of essential alternatives) *For all* $A, B \in \Pi(X)$, $\big[A \succeq B$ *if and only if* $\#[\max(A) - A^B] \geqslant [\#\max(B) - B^A]\big]$.

Proposition 3 (Pattanaik et Xu (1998)) *The relation* $\succeq$ *satisfies simple dominance (condition 9), simple non-dominance (condition 8), I-monotonicity (condition 5), type 1 irrelevance of dominated alternatives (condition 6), type 2 irrelevance of dominated alternatives (condition 10) and weak composition (condition 11) if and only if* $\succeq$ *is the weighted cardinal ranking of essential alternatives.*

Lexicographic version of the cardinality rule Another way to avoid the problem raised by the example displayed above is given by the lexicographic approach of the cardinal ranking. For instance, situations where some state-newspapers are available may still sound better than one with some free-press available. Romero-Medina (2001: 185) propose to 'sequentially remove the first element in all the reasonable persons' preferences and compare the available sets of newspapers according to this new set of preferences. This new reference set where alternatives are sequentially eliminated [...] is a compromised idea of freedom that can only be justified when the set of reasonable preferences cannot discriminate'.

2.3 Diversity and exercice of significant choices

The independence condition (see condition 3) used in the cardinal ranking (rule 1) is likely to raise some problems, well illustrated by this example (see Pattanaik et Xu (1990: 389)).

Let's take the following universal set of options: {train, car 1, car 2}. By indifference, we get: {train} $\sim$ {car 1}. By independence, we then get: {train, car 2} $\sim$ {car 1, car 2}, meaning that these two sets offers exactly the same freedom of choice. Appealing to intuition, this result raises substantial problems. The first set allows for a choice among a wide scope of diverse transportation modes. The choice of options among the first set is then harder and more substantial because choosing train rather than car expresses the kind of life you decided to lead. In the second set, the act of choosing just exercises on the kind of car, e.g. the color of the car, which is less significant. If we imagine that this second car is exactly similar in brand, type, series, color..., this result is even more annoying. It does contradict with the idea of valuing for themselves significant choices. To express through the ranking of sets the idea of significant choices, the independence condition has to be weakened so that diversity or similarity of options are now taken into account.

Speaking of significant choices refer to two kinds of situations: according to P. Jones and R. Sugden (1982, p. 57), 'an option is not significant in relation to a choice set if it is either indistinguishable from another option or ineligible.' We choose here to save the expression of 'significant choice' for the first case : making a choice between indistinguishable options is not significant. The more diverse the options in the set, the more significant is the choice of options from the set. Indeed, diversity induces a harder choice, meaning the exercise of choice is more meaningful. J. S. Mill values significant choices and, through it, the exercise of choice in itself, whichever it is. It is through the act of choice that human faculties are developing: 'The human faculties of perception, judgment, discriminative feeling, mental activity, and even moral preference, are exercised only in making a choice. [...] The mental and moral, like the muscular powers, are improved only by being used' (Mill (1859: 74-75)). And these faculties are one of the elements of well-being (see Mill (1859: 72)).

Therefore, the issue at stake becomes the way to introduce the notion of similarity or diversity of options. Of course, this requires at least to give some more information about these options. But as far as indifference condition is not challenged at this point (See below), wide information to describe the option, e.g. the attributes of options, is not necessary. The only required information concerns exclusively whether or not options are similar, or, at most, how diverse they are. This will affect the very definition of monotonicity, in the same way that the account for reasonable preferences affected autonomy orderings, as well as the formulation of indifference as well. Note that some propositions (See Rosenbaum (2000), Nehring and Puppe (2002)) use more information than necessary for this specific purpose, describing the attributes of the options. It actually mean they endorse another notion of freedom as well, that we will present below.

Different ways are explored to characterize similarity of options: either we focus on the set in itself; either we focus on the options themselves. In the first case, the diversity of the set depends on its extreme options. In the second case, each option from the set is considered as likely to be similar or not (binary or vague judgments of similarity), nay more or less similar (cardinal or ordinal degrees of similarity).

2.3.1 Extreme alternatives

A first way to take into account diversity is to focus on the most extreme alternatives in the opportunity set.

M. Klemish-Alhert (1993) represents opportunity sets as convex hulls in the space of universal set of goods. The conditions used to compare sets are re-expressed in this framework. Convex hull monotonicity for instance is a condition allowing to value the scope of alternatives in the hull. This hull will be larger if its boundary alternatives will be far from one another.

E. Rosenbaum (2000) assesses freedom of choice from the scope of choices according to certain characteristics, requiring a wide amount of information at this point. Formally, freedom of choice is a function of the mathematical distance between extreme points in the space of characteristics. The more distinct these characteristics, the more free the individual facing the set. If some characteristic reveal more important than others, this unequal importance is expressed by unequal weight for this characteristic.

We are then led to focus on the extreme positions. M. Klemish-Alhert (1993: 197) finds this result controversial because it is likely to induce undesirable consequences. For instance, enjoying freedom in a country where it is just allowed to express extreme opinions will be considered, in this framework, as more valuable than in any country where all opinions can be expressed except extreme ones. To avoid these counter-intuitive consequences, it seems interesting to take into account how each option contribute to the diversity of the set, rather than focusing on extreme options.

2.3.2 Binary judgements of similarity

Diversity or similarity of options can be just a question of binary judgments : either two options are similar, either they are not.

P. Pattanaik and Y. Xu (2000) introduce on a similarity relation, written S, and defined over X is reflexive and symmetric (but not supposed transitive). We read xSy as: 'x is similar to y' and $\neg xSy$ as: 'x is not similar to y'. For all $A \in \Pi(X)$, we say that A is homogeneous if and only if, for all $a, a' \in A$, $a'Sa$. For all $A \in \Pi(X)$, a similarity based partition of A is defined as a class $\{A_1, ..., A_m\}$ such that: (1) $A_1, ..., A_m$ are all non-empty subsets of A; (2) $A_1 \cup ... \cup A_m = A$; (3) $A_A, ..., A_m$ are pairwise disjoint; and (4) for all $k \in \{1, ..;, m\}$, A_k is homogeneous. The similarity based partition will be denoted by $\phi(A), \phi'(A), \phi"(A)$, etc.

Monotonicity and composition are re-formulated in this new setting.

Condition 12 (S-monotonicity) *For all $A \in \Pi(X)$ such that A just contains similar alternatives according to S, and, for all $x \in X \setminus A, [xSA \Rightarrow A \cup \{x\} \sim A]$ and $[\neg xSA \Rightarrow A \cup \{x\} \succ A]$.*

Condition 13 (S-composition) *For all $A, B, C, D \in \Pi(X)$, if $[A \cup C = B \cup D = \emptyset$, C and D contains similar alternatives, and C is not similar to $A]$, then $[(A \succeq B$ and $C \succeq D)$ implies $A \cup C \succeq B \cup D]$ and $[(A \succ B$ and $C \succ D)$ implies $A \cup C \succ B \cup D]$.*

Under the simple similarity based rule, opportunity sets are ranked according to the cardinalities of their smallest similarity-based partitions.

Rule 5 (Simple similarity based ordering) *For all $A, B \in \Pi(X)$, $A \succeq_{\#S} B$ if and only if $\#\phi(A) \geq \#\phi(B)$.*

Proposition 4 (Pattanaik and Xu (2000)) *$\succeq$ satisfies indifférence between no-choice situations (condition 1), S-monotonie (condition 12) and S-composition (condition 13) if and only if $\succeq$ is the simple similarity based ordering.*

This result answers the objection raised against the cardinal ordering, in which freedom of choice is growing even in the case clones are added to existing alternatives.

2.3.3 Approximations of similarity

It might be difficult to definitely hold that 'alternatives are similar' or 'they are not'. S. Bavetta and M. Del Seta (2001) use the concept of rough approximations of the sets . They consider, in the universal set of options, first, the inner (or lower) set of options that is just composed by similar options, and second, the outer (or upper) set of options, whose complement do not include any options that are similar to options from the set. This induces two kinds of possible orderings, based on the cardinality of each of these rough approximations of the sets.

2.3.4 Assessing degrees of similarity

Next step is taking into account degrees of similarity rather than binary judgements about similarity. M. van Hees (2004) raises a problem linked with this ambition. He establishes that the characterization of similarity based orderings is often impossible because of the definition of distance between alternatives and sets.

S. Bervoets and N. Gravel (2003) propose an opportunity sets ordering based on an ordinal notion of diversity, when W. Bossert, P. Pattanaik and Y. Xu (2002) propose a cardinal approach of diversity. The study of biodiversity, run by K. Nehring and C. Puppe (2002), illustrates another way to take into account the problem of diversity, considering the diversity of attributes characterizing options.

Another way to capture the valuation of diversity is proposed by F. Gaspart and myself (2005)[3]. They observe that it is easy to choose between these two alternatives: working and be well-paid on the one side and not-working and starving on the other side. But is is not as easy to choose between these two ones: working and be paid, not working and be a little paid. In the second opportunity set, exercising a choice identifies the kind of life the person is really valuing while a reasonable person is more likely to just reveal she does want to starve in the first one. A. Baujard and F. Gaspart capture the idea of valuing hard choice in particular through a specific trade-off between utility and choice. Diversity is taken into account from a trigger value where utility and diversity seem indifferent.

2.4 Identifying constraints and negative freedom

Freedom is about the absence of constraints. Different conceptions of freedom corresponds to the focus on different kinds of constraints. ‘Negative freedom’ refers to the absence of coercion (See I. Berlin (1969)). As a relational concept, freedom is defined relatively to the constraints that other individuals impose to individual choices. It is a space in which individual can act and choose without being prevented by others. In this sense, freedom, or unfreedom, does not depend on self-abilities of the individual to do what he wants, but on other people’s intentional meddling, coercion or oppression. Measuring negative freedom supposes to measure the absence of this kind of constraints, rather than any other constraints. The more preventions there are, the less freedom.

If freedom were to be defined by this only criterium, then, two situations would be indifferent if there were no coercion in both but many available options in the first one when just one option is available in the second one. As a counterpoint to this, freedom is higher when non-prevented opportunities are more numerous. Measuring negative freedom therefore has then two aspects: the identification of the origin of constraints and the situation of opportunity.

Constraints may be of different kinds. An action is an opportunity if it is doable. But the reason why it is doable is not just that there is no coercion against it realization. Twenty years ago, people were not prevented from using cellular phones but they did not and could not use them because it was just technologically impossible. This opportunity did not exist. This non-existence did not affect negative freedom but the actual ability to use the cellular phone. It sounds very clarifying to actually distinguish between different kinds of constraints to focus on the negative aspect of freedom when assessing opportunity sets, e.g. firstly, technical or social constraints, and, secondly, external constraints.

Following van Hees (1998)[43], an opportunity situation consists of the ordered pair of a feasible set and an opportunity set, (A, G), describing the set of actions A doable from the fact that there exists an adapted technology, and the opportunity set G composed by actions that nobody is preventing us from doing by coercion, and that we will call by the general term 'external constraints'.

2.4.1 External constraints and negative freedom

In a first approach, we focus on the external constraints, considering all actions are possible, as far as technological possibilities are concerned. The basic conditions to capture the idea of freedom are then re-formulated in the framework of opportunity situations.

Condition 14 (Strict monotonicity) *$\forall x, y \in A$, $x \neq y$, $(A, \{x, y\}) \succ (A, \{y\})$.*

Condition 15 (Indifference between no-choice situations) *$\forall x, y \in A$, $(A, \{x\}) \sim (A, \{y\})$.*

The next condition capture the role of external constraints in the definition of negative freedom. A person's negative freedom depends on the things she is not free to choose. A situation in which an option has become technologically feasible and can be chosen without constraints yields the same amount of freedom as the one in which it was not yet feasible. In others words, as soon as there is not further external constraints, there is no reason why negative freedom should change.

Condition 16 (Immunity to opportunities deriving from new technology) *$\forall G \in \Pi(X)$ et $\forall x \in X$: $(A, G) \sim (A \cup \{x\}, G \cup \{x\})$.*

Rule 6 (Constraints-based cardinality ordering) *$(A, G) \succeq (B, F)$ if and only if $\#(B - F) \geqslant \#(A - G)$.*

According to the constraints based cardinality quasi-ordering, the less external constraints to realize technically possible actions, the more free.

Proposition 5 (Van Hees (1998)) *Preorder $\succeq$ satisfies indifference between no-choice situations (condition 15), strict monotonicity (condition 14), and immunity to opportunities deriving from new technology (condition 16) if and only if $\succeq$ is the constraints based cardinality quasi-ordering.*

2.4.2 Technological constraints and negative freedom

Even though technological constraints may be independent from external constraints, they may affect the actual negative freedom derived from an opportunity situations. As a matter of fact, if some new technology now exist but if it does not change the opportunity set, then the freedom derived from the set is reduced. Innovations are likely to increase freedom to the only condition that it is available to the person.

Condition 17 (Decreasing with new technology) $\forall G \in \Pi(X)$ *and* $\forall x \in X - A \ : (A, G) \succ (A \cup \{x\}, G)$.

As in condition 16, both the feasible set and the opportunity set is changing. The opportunity situation resulting from the combination of two should be ranked exactly in between the two situations of which it is the combination. This captures the fact that the increase in available options does not necessarily outweigh the increase in the number of forbidden options.

Condition 18 (Independence with variable technology) *If A and B are disjoint, then* $\forall G, F \in \Pi(X)$:
$(A, G) \sim (B, F) \Rightarrow (A, G) \sim (A \cup B, G \cup F) \sim (B, F)$
and $(A, G) \succ (B, F) \Rightarrow (A, G) \succ (A \cup B, G \cup F) \succ (B, F)$.

Condition 19 generalize indifference between no-choice situations by considering both the feasible set and the opportunity set rather than just the allowed alternatives.

Condition 19 (Neutrality to any permutation) $\forall G \in \Pi(X)$ *and for any permutation π of X, $(A, G) \sim (\pi A, \pi G)$, where πA and πG stand for the images of A and G under π, respectively.*

The ordering proposed by Steiner (1983) depends, not any more on the difference, but on the ratio between opportunity set cardinality sets of doable actions.

Rule 7 (Steiner's ordering) $(A, E) \succeq_* (B, F)$ *if and only if* $\dfrac{\#E}{\#A} \geqslant \dfrac{\#F}{\#B}$.

When technological innovations increase opportunity set, freedom is increasing.

Proposition 6 (Van Hees (1998)) *Preorder $\succeq$ satisfies decreasing with new technology (condition 17), independence with variable technology (condition 18), and neutrality to any permutation (condition 19) if and only if $\succeq$ is the Steiner's ordering.*

2.5 Contribution to actual realizations and positive freedom

Another conception of freedom, opposed to negative freedom, is positive freedom (See Berlin (1969)). It is not any more so important to identify the kind of constraints to do any actions. On the contrary, the substantial part of freedom stands in the actual possibility or impossibility to do them. The ordering of opportunity sets of options to capture the idea of actual freedom is to be distinguished from those to capture the idea of freedom of choice.

On the one side, indifference condition, that was a relevant requirement for freedom of choice *per se*, raises in itself a disturbing problem. With indifference, we should consider : $\{$ education $\} \sim \{$ a glass of champagne $\}$. Yet, the first option is driving to many different, valuable, and lasting opportunities (such as jobs and consumption going along, pleasure derived from culture...), while the second one provides a short time of pleasure and does not open, eventually, many other opportunities. In other words, to speak about actual freedoms, it is important to give up the assumption, imposed by condition 1, according to which options are considered all homogeneous. Two ways are then possible to differentiate between options. The first way consists in considering the actual utility or welfare in general provided by each options (See Sen (1990: 470)). In this case, the obtained ranking captures the idea of utility or global well-being displayed below. The second way consists in considering that freedom derives from the number of actions that are eventually possible to achieve (See Carter (1999)): the idea of overall freedom is expressed in this case.

On the other side, it is important to incorporate the impact of the constraints on the ability to take advantages of the options. People are not prevented by anyone to buy some expensive jewelry place Vendôme, but most of them could never afford it, even though they would invest their whole budget in it. It reveals important to describe the systems of constraints, whatever their nature, limiting the scope of doable actions (See current work by Bavetta, XXX). Besides, McEnroe and Becker playing together can both win, but not all together. They do not have the collective freedom to both win the match (See Carter (1999: 258)). It is then important to take into account the system of individual interactions affecting this actual freedom.

2.6 Maximum options and utility

The issue at stake in this subsection is the way of representing utility in a ranking opportunity sets framework. Opportunity set may not induce utility *per se*. According to the experience requirement (see Glover (1977)[16], Griffin (1986), Haslett (1990)), an individual should experience the utility of options, while he will just experience one option among the set. This means that an opportunity set is not an object of utility in itself, it can just be the object of indirect utility. So if the retained criterium to rank the sets is utility, what we will be studying is the indirect utility of opportunity sets.

The indirect utility derived from the set will depend on the satisfaction the chosen option will provide. We then have to imagine which option might be chosen from the set. This will depend on the relevant preference relation: the actual preference relation when it is known and fixes, the probable preference relations when they are flexible.

2.6.1 When preferences are given

In a narrow welfarist point of view, choice has no intrinsic value. Comparing two sets amounts to comparing the utility of the sets, which is captured by the dominance condition:

Condition 20 (Simple dominance) *For all* $x, y \in X$*, if* $x\mathbf{P}y$*, then* $\{x\} \succ \{y\}$

Opportunity sets, rather than singletons, have some indirect value because the choice among a wider set might open some better options. This idea is captured by the weak monotonicity condition.

Condition 21 (Weak monotonicity) *For all* $A, B \in \Pi(X)$*,* $A \supseteq B \Rightarrow A \succeq B$

When these conditions hold and preferences (or utility) are well-defined, the following ordering allows to compare opportunity sets.

Rule 8 (Rankings of best elements) *For all* $A, B \in \Pi(X)$, $A \succeq B$ *if and only if* $\max\{U(x)|x \in A\} \geq \max\{U(x)|x \in B\}$.
We can also write this rule the following way : $A \succeq B$ *if and only if, for all* $x_B \in B$*, there exists* $x_A \in A$ *such that* $x_A \mathbf{R} x_B$.

This indirect-utility based ranking makes sense if, first, options from the set do describe everything relevant for utility, and, second, the used preference relation (or utility function) is the preference used for the actual choice among this set. But preferences are likely to change. When they are not given and certain, the range of future preferences, relevant for the actual choice, have to be identified.

2.6.2 When preferences are risky

Saying that preferences are risky means that there is no unique individual preference relation. Individual preferences depend on a parameter whose probability distribution is well-known. K. Arrow (1995) proposes an ordering based on this principle[6].

2.6.3 When preferences are uncertain

Let's say now that preferences are neither given, neither risky. As we do not know the probability attributed to any imaginable preference relation, the situation at stake is characterized by risk but by uncertainty. Assessing the indirect utility from a set supposes to take into account a scope of different preference relation while it is not possible to identify the one that will reveal eventually relevant. D. Kreps (1979) proposes a ranking that captures the idea of preference for flexibility.

He formulates a composition condition.

Condition 22 (Composition) $A \sim A \cup B$ *implies that, for all* C, $A \cup C \sim A \cup B \cup C$.

Kreps shows that we may compare two sets by comparing the value of the following function $v(.)$ of the sets: $v(A) := \sum_s \pi(s).[max_{x \in A} U(x,s)]$, where s is a positive random variable and π is the subjective probability measure π. This function is then used to order opportunity sets according to $\succeq$. It can be shown that it satisfies weak monotonicity (condition 21) and composition

[6]It is interesting to note there is no need here to use a different framework than the traditional microeconomic framework for utility.

(condition 22). The maximal elements of the sets according to each preference (or utility function) is the only important information to compare sets.

But, in this formulation, the probability distribution does not have real significance and the additive form is not necessary neither. It is then possible to propose an ordinal version of this ordering :

Proposition 7 *If X is finite, a binary relation $\succeq$ on $\Pi(X)$ is complete and transitive and satisfies weak monotonicity (condition 21) and composition (condition 22) if and only if there exist a finite set S, a function $U : X \times S \to \mathbb{R}$, and a strictly increasing function $u : \mathbb{R}^S \to \mathbb{R}$ such that, if $w : \Pi(X) \to \mathbb{R}^S$ is defined by: $(w(A))(s) := \max_{A \in \Pi(X)} U(A, s)$,*
then, $u \circ w(: \Pi(X) \to \mathbb{R})$ represents $\succeq$.

Besides, J. Foster (1993) suggests that 'a set provides more effective freedom if the set is preferred meanwhile by all possible preference relation'. As we do not know which preference will be at stake or what probability is assigned to each preference, just the scope of possible preferences, we may consider this situation as one of uncertainty. We would reinterpret his ranking the following way: 'a set provides more indirect utility in uncertainty over preferences if and only if all preferences the individual may have over options would drive him to choose this set'.

Of course, if the possible preferences are now interpreted as reasonable preferences an individual may have rather that actual preferences an individual is likely to have, the right interpretation of the obtained ordering is not about flexibility and indirect utility but about autonomy, as we have seen above.

3 Rankings of opportunity sets according to several prudential values

We have presented above different rankings according to some specific and unique prudential value. The objective of constructing global well-being rankings requires this process of combination of different values in a single ranking (See Griffin (1986) for a thorough discussion of well-being and its determinants). The values are stake could be either of the ones we displayed above. We will though focus on freedom of choice and utility, as these are more represented in the literature. The differences between the rankings of global well-being therefore do not stand in the 'ingredients" but rather on the kind of retained combination, namely the nature of commensurability between the values (See Griffin (1986: 75-92)): weighting values, lexical order of the values, equal consideration of different values, or discontinuity.

3.1 Weighting values

Weighting values supposes that a trade-off between values is meaningful and possible. There therefore exists a common scale for values. In others words, a superior value constitutes the common scale based on the different prudential values to judge the situations. This one value may be called well-being or utility (See Mill (1866: book VI, ch. XII, § 7)). Judgements based on weighting different values are then eventually welfarist, though utility taken as the satisfaction of preferences might not be the only value at stake. That is the reason why we will rather use the term 'global well-being': global well-being is the substantial value depending both on utility and freedom of choice.

There is two ways to express this actual welfarism: either the two values are linked by utility, either the only information at stake from each situation is utility.

3.1.1 Values bound to utility

C. Puppe (1996) suggests to take into account both utility, through monotonicity with respect to set inclusion (condition 4) and independance of non-essential alternatives (condition 23) and freedom of choice, through preference for freedom of choice (condition 24), though freedom of choice is important only as far as it is freedom to choose useful options. Thus, even though it is said that the two values are at stake, just one value is eventually substantial: utility.

As we said and defined before, for a given situation and for given individual preferences, a set A provides more freedom with alternative $\{x\}$ if and only if this is an essential alternative. The set of all essential alternatives of set A is written $E(A)$. We have : $E(A) \supseteq A, \forall A \in \Pi(X)$.

According to independence of non-essential alternatives condition, the freedom derived from a set just depends on its essential alternatives. It is then equivalent to face a set or the set of its essential alternatives. This allows to capture the idea of domination and the importance of utility in the ranking.

Condition 23 (Independence of non-essential alternatives) *$\forall A \in \Pi(X)$, $E(A) \sim A$.*

To capture the idea of valuing autonomy, we should set: $\forall x \in A, A \succ A \setminus \{x\}$. Our intuition is that, in all set, there exists at least one option that deserves consideration : $E(A) \neq \emptyset$. In condition 24 on the contrary, A is preferred to $A \setminus \{x\}$ just because it includes this (at leats) one valuable option according to the individual preference. What explains the value of set is the value of options, rather than the structure of the set as in autonomy or freedom of choice *per se* rankings. Even though this condition seems to attach value to freedom of choice, the latter is actually just taken into account for its contribution to indirect utility.

Condition 24 (Preference for freedom of choice) $\forall A \in \Pi(X)$, $\exists x \in A$ *such that* $A \succ A \setminus \{x\}$.

Sets including essential alternatives are preferred.

Rule 9 (Domination relation of essential alternatives) $\forall A, B \in \Pi(X)$, $A \succeq B$ *if and only if* $E(A \cup B) \subseteq A$.

Proposition 8 (Puppe (1996)) *Preorder $\succeq$ satisfies preference for freedom of choice (condition 24), weak monotonicity (condition 4), independence of non-essential alternatives (condition 23) if and only if $\succeq$ is the domination relation of essential alternatives.*

3.1.2 A welfarist evaluation

W. Bossert (1997) proposes some rankings based on overall well-being. Different values may contribute to the level of individual well-being, such as utility and freedom of choice, but we just consider the instrumental value of these prudential values.

All possible utility functions are taken into account. For each of them, there exists a ranking of opportunity sets based on utility of the alternatives of the set. Then, the monotonicity condition should not necessarily hold. A minimal indifference condition rather seems more acceptable: there exists a overall well-being function and an option, such that adding it to the set does not necessarily increase the overall well-being derived from the set. The relation does indeed depend on the overall well-being of the set rather than on the utility of options. $\mathcal{R}$ denotes the set of all orderings on $\Pi(X)$. An opportunity set ranking rule assigns an element of $\mathcal{R}$ to each utility function $U \in \mathcal{U}$. Formally, an opportunity set ranking rule is a mapping: $R : \mathcal{U} \mapsto \mathcal{R}$. For simplicity, we will write: $\mathbf{R}_U = F(U)$, with $\mathbf{I}_U$ and $\mathbf{P}_U$ for the symmetric and asymmetric part of $\mathbf{R}_U$.

Condition 25 (Minimal indifference) $\exists U \in \mathcal{U}, x \in X$, *and* $y \in X \setminus \{x\}$ *such that* $\{x, y\}\mathbf{I}_u\{x\}$.

This condition, as the others, may be expressed with preference relations rather than with utility functions: $\exists \succeq$ and $x \in X, Y \in X \setminus \{x\}$ such that $\{x, y\} \sim \{x\}$.

It is then possible to identify the underlying utility functions to capture the role of utility, with neutrality and extension conditions. The former reduces information needed to rank opportunity sets to the maximal possible utility for each set. We are then in a extended welfarist framework (capturing overall well-being), though not really in an indirect utility framework (capturing utility). Extension condition sets that rankings of set should respect individual preferences over options when the sets are reduced to singletons. This amounts to, though in a different framework to the simple dominance condition 32.

Condition 26 (Extension) $\exists U \in \mathcal{U}, \forall x, y \in X, \{x\}\mathbf{R}_u\{y\}, U(x) \geq U(y)$.

Independence condition captures the fact that rankings of two sets remains still if their common elements are removed from both sets. This is basically a separability condition.

Condition 27 (Separability) $\forall \mathbf{R} \in \wp, \forall A, B \in \Pi(X), \forall x \in X \backslash (A \cup B)$, $A\mathbf{R}_U B \Leftrightarrow (A \cup \{x\})\mathbf{R}_U(B \cup \{x\})$.

W. Bossert introduces a continuity condition, interpreted as a regularity condition :

Condition 28 (Regularity) $\forall n \in \mathbb{N}, \forall u \in \mathbb{R}^n, \forall A, B \in \Pi(X)$, *such that* $|A| = |B| = n$, *the sets* $\{v \in \mathbb{R}^n \mid U[A] = u$ *and* $U[B] = v$ *and* AR_uB *for some* $U \in \mathcal{U}\}$ *and* $\{v \in \mathbb{R}^n \mid U[A] = u$ *and* $U[B] = v$ *et* BR_uA *for some* $U \in \mathcal{U}\}$ *are closed.*

Rule 10 (Anonymous ranking of all elements from the set) *There exists a function* $g : \mathbb{R} \to \mathbb{R}$, *and* $t \in \mathbb{R}$, *such that* $g(t) = 0$. $A \succeq B$ *if and only if* $\forall n, m \in \mathbb{N}, \sum_{i=1}^{i=n} g(v(A_i)) \geqslant \sum_{i=1}^{i=m} g(v(B_i))$, *where* $v(A_i) = (u(a_1), ...u(a_i))$.

The ranking of the set is based on the sum of utilities derived from each option included in the set.

Proposition 9 (Bossert (1997)) *Preoder* $\succeq$ *satisfies minimal indifference (condition 25), regularity (condition 28), extension (condition 26) and separability (condition 27) if and only if* $\succeq$ *is the anonymous ranking of all elements of the set.*

It is besides possible to study the impact of modifying information requirements of utility on these rankings. Different measuring assumptions induced different rankings, varying along with parameters (See Bossert (1997: 107–110)). For instance, if we assume a cardinal measurability, it is possible to assign different weight to each option according to their desirability.

3.2 Lexical order of the values

Trumping introduces a weak form of incommensurability between values. It allows comparability, but one value outrank the others as strongly as possible. We then talk of lexical priority of one value over the other. All distinct value may be assigned an intrinsic importance then, but one of them reveals more substantial. The secondary values affects the ranking in the only case where the first one is already satisfied. In our framework, this amounts to give a lexical priority of utility over freedom, or on the contrary, a lexical priority of freedom over utility.

3.2.1 Priority of utility over freedom

First, let's express priority of utility over freedom

Condition 29 (Indirect preference) $\forall A \in \Pi(X)$, *with* $\#A > 1$, $\{\max(A)\} \succ \{A \setminus \max(A)\}$

The weak independence condition recalls the independence condition (condition3), according to which the ranking of sets is the same for sets on the one side, and these sets where we added new options. In this weaker version of independence, we assume that this added element is not strictly preferred to any best elements from the previous sets.

Condition 30 (Weak independence) $\forall A \in \Pi(X)$, $\forall x \in X \backslash (A \cup B)$, $[\max(A)\mathbf{P}x$ *and* $\max(B)\mathbf{P}x] \Rightarrow [A \cup \{x\} \succeq B \cup \{x\} \Leftrightarrow A \succeq B]$

With to the preference-first lexicographic relation, the sets are ordered first by ranking their maxima according the the individual actual preference relation. In case of ties, they are ranked according to the cardinality rule.

Rule 11 (Preference-first lexicographic relation) *For all* $A, B \in \Pi(X)$, $[A \succeq B$ *if and only if* $(\max(A) P \max(B))$ *or* $(\max(A) = \max(B)$ *and* $\#A \geqslant \#B)]$.

Proposition 10 (Bossert, Pattanaik and Xu (1994)) *Quasi-ordering* $\succeq$ *satisfies simple monotonicity (condition 33), simple indirect indifference (condition 34), weak independence (condition 30), and indirect preference (condition 29) if and only if* $\succeq$ *is the preference-first lexicographic relation.*

3.2.2 Priority of freedom over utility

It is possible to capture the converse priority, where freedom of choice becomes the prior value over utility. It is just in case the sets are indifferent in terms of freedom that utility plays a role to order them.

Condition 31 (Simple priority to freedom) $\forall$ *distincts* $x, y, z \in X$, $x\mathbf{P}y\mathbf{P}z \Rightarrow \{x, z\}\mathbf{P}\{x\}$

According to simple priority to freedom over utility condition, a set with two elements is preferred to a singleton equal to the best element. This unequivocally captures the substantial priority of freedom over indirect utility.

Rule 12 (Cardinality-first lexicographic relation) *For all* $A, B \in \Pi(X)$, $[A \succeq B$ *if and only if* $(\#A \geqslant \#B)$ *or* $(\#A = \#B$ *and* $\max(A)\mathbf{R}\max(B))]$

The sets are ordered according to their cardinality. In case of ties, they are ordered according to the ranking of their best elements.

Proposition 11 (Bossert, Pattanaik and Xu (1994)) *Quasi-ordering* $\succeq$ *satisfies simple dominance (condition 32), simple indirect indifference (condition 34), weak independence (condition* **??***), and simple priority to freedom (condition 31) if and only if* $\succeq$ *is the cardinality-first lexicographic relation.*

3.3 Equal consideration of different values

Pluralism of value a priori requires to consider different values without any priority neither weighting (See, on the contrary, the link between monism and trade-off of values above). Freedom of choice and utility are both and meanwhile considered for their intrinsic value. They are considered *incommensurable*. This induces two kinds of results: either incompleteness, either impossibilities.

3.3.1 Incompleteness

Putting two values on the same plane means that both of them need to be satisfied to make a judgement. Whenever one value contradicts the other, no conclusion can be drawn.

Condition 32 (Simple dominance) $\forall x, y \in X, x\mathbf{P}y \Rightarrow \{x\} \succ \{y\}$

Simple dominance captures the intrinsic value of utility.

Condition 33 (Simple monotonicity) $\forall x, y \in X$ *such that* $x \neq y$, $\{x, y\} \sim \{x\}$.

According to the simple indirect indifference condition, the best element always play a dominant role in ranking two elements sets.

Condition 34 (Simple indirect indifference) $\forall$ *distinct* $x, y, z \in X$, $x\mathbf{P}y\mathbf{P}z \Rightarrow \{x, y\} \sim \{x, z\}$.

The weak indirect preference condition implicitly assign a priority to considerations of the individual's preferences over considerations of freedom.

Condition 35 (Weak indirect preference) $\forall A \in \Pi(X)$ *with* $\#A > 1$, $\neg\left(A\backslash\{\max(A)\} \succ \{\max(A)\}\right)$

According to simple non-comparability condition, there is no trade-offs between utility and freedom in simple comparisons. Let ∞ denote the non-comparability relation associated with $\succeq$.

Condition 36 (Simple freedom-utility noncomparability) $\forall x, y, z \in X$ *distincts*, $x\mathbf{P}y\mathbf{P}z \Leftrightarrow \{x\}\infty\{y, z\}$

The dominance relation requires that the two conditions, cardinality and indirect utility, are both satisfied, though we can very well imagine numerous cases where these two conditions do not meet. In this latter case, the result characterizes incompleteness.

Rule 13 (Dominance relation) $\forall A, B \in \Pi(X), A \succeq B$ *if and only if* $(\max(A)\mathbf{R}\max(B)$ *and* $\#A \geqslant \#B)$

Proposition 12 (Bossert, Pattanaik et Xu (1994)) *Preorder $\succeq$ satisfies simple dominance (condition 9), simple monotonicity (condition 33), simple indirect indifference (condition 34), weak independence (condition 30), weak indirect preference (condition 35), and simple freedom-utility noncomparability (condition 36) if and only if $\succeq$ is the dominance relation.*

3.3.2 Impossibilities

A second way to capture equal consideration of freedom and utility is to require they are both satisfied. But this induces impossibility results.

According to the preference basedness condition, a set in which an preferred option has been substituted to another one will always be preferred. This condition captures the role of utility in ranking opportunity sets.

Condition 37 (Preference basedness) *$\forall A \in \Pi(X)$, $x \in X \setminus A$, $y \in A$, $[x\mathbf{R}y$ if and only if $(A \setminus \{y\}) \cup \{x\} \succeq A]$.*

The minimal preference for freedom condition is a weaker version of the simple monotonicity condition. It captures the fact that freedom has some importance in itself.

Condition 38 (Minimal preference for freedom) *For all $A \in \Pi(X)$ with $\sharp A \geq 2$, $\exists B \subseteq A$ such that $A \succ B$.*

Let the elements of X be $x^i, i = 1, ..., n$. For each $i = 1, ..., n$, let $(x^i_j)_{j \in \mathbf{N}}$ be a sequence converging to x^i. According to the continuity conditions, a small perturbation of the alternatives in a given set of opportunities should only have a small impact on the ranking of the set *vis-à-vis* all other sets.

Condition 39 (Continuity of $\succeq$) *$\forall A \in \Pi(X)$,*
$[\forall j \in \mathbf{N} : \{x^1_j, ..., x^n_j\} \succeq A] \Rightarrow \{x^1, ..., x^n\} \succeq A$
and
$[\forall j \in \mathbf{N} : A \succeq \{x^1_j, ..., x^n_j\}] \Rightarrow A \succeq \{x^1, ..., x^n\}$

Condition 40 (Continuity of $\succ$) *$\forall A \in \Pi(X)$,*
$\{x^1_j, ..., x^n_j\} \succ A \Rightarrow [\exists j_0 \forall j \geq j_0 : \{x^1, ..., x^n\} \succ A]$
and
$A \succ \{x^1_j, ..., x^n_j\} \Rightarrow [\exists j_0 \forall j \geq j_0 : A \succ \{x^1, ..., x^n\}]$

Proposition 13 (Puppe (1995)) *There does not exist any preorder $\succeq$ such that $\succeq$ satisfies preference basedness (condition 37), minimal preference for freedom (condition 24), and continuity (conditions 39 and 40).*

N. Gravel (1994), (1998) did establish some similar impossibility results in this framework.

3.4 Discontinuity

We have presented above two extreme positions. Perfect commensurability, through weighting, amounts to one value-based ranking. Besides, total incommensurability, through trumping or equal consideration of values, allows to attach some intrinsic importance to both values, but it is so demanding that it induces either incompleteness, impossibilities or gives eventually too much importance to one of the two values. These extremes are disturbing and we would appreciate to capture the idea that both values have some intrinsic importance without dramatic inequalities of treatment and, yet, allows some general rankings. We want to find something between trumping and weighting values.

J. Griffin (1986: 85–89) does propose a way out: discontinuity. He takes the example of commensurability between prosperity and liberty: 'when we see that liberty is not, in the strict sense, lexically prior to prosperity, it is natural to retreat to the view that, if prosperity is assured to some minimal level, then the priority holds. But [...] Liberty, as important it is, is not such a heavyweight among prudential values that, even in the limited domain we have defined, it is bound to trump any other value capable of achieving fairly heavyweight status itself. The mistakes here seems to be to think that certain values – liberty for instance – as types outrank other values – prosperity for instance. Since values, as types, can vary greatly in weight from token to token, it would be surprising to find this kind of discontinuity at the type – or at least at a fairly abstract type – level.'

In other words, we do not consider wether the values may be satisfied or not in some specific situations. We rather consider them to be more or less satisfied, information that is specifically derived from the rankings. This means that there is a way to measure how satisfied each of the values is. With discontinuity, a certain quantity of one value, let's say utility, is necessary to be able to say that, after that point, the other value, let's say freedom, becomes more valuable. In others words, it is necessary to attain trigger value for each of them to consider the importance of the other (conversing lexical order) or that the other may now outranked the first one (conversing weights).

Baujard and Gaspart (2004), in a economic environnement framework, have used such a notion of discontinuity to capture the idea of a specific commensurability between utility and freedom, taken as the value of the exercice of significant hard choices.

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